International Journal of Theoretical Physics, Vol. 46, No. 3, March 2007 (© 2007)
DOI: 10.1007/s10773-006-9108-1

Gravitational Wave Holography

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Received September 14, 2005; accepted March 5, 2006
Published Online: January 31, 2007

We represent and discuss a theory of gravitational holography in which all the involved waves; subject, reference and illuminator are gravitational waves (GW). Although these waves are so weak that no terrestrial experimental set-ups, even the large LIGO, VIRGO, GEO and TAMA facilities, were able up to now to directly detect them they are, nevertheless, known under certain conditions (such as very small wavelengths) to be almost indistinguishable (see P. 962, in Misner, C. W., Thorne, K. S., and Wheeler, J. A. (1973). *Gravitation*, Freeman, San Francisco.) from their analogue electromagnetic waves (EMW). We, therefore theoretically, show, using the known methods of optical holography and taking into account the very peculiar nature of GW, that it is also possible to reconstruct subject gravitational waves.

KEY WORDS: holography; gravitational wave; interference.

PACS numbers: 42.40.-i, 42.40.Eq, 04.30.-w, 04.30.Nk

1. INTRODUCTION

The theory of electromagnetic (optical light, X-rays, γ-rays) and matter (electrons, atoms) holography is well established (see, for example, Barton, 1988; Collier *et al.*, 1971; Gabor, 1948, 1949,1951; Han *et al.*, 1991; Harp *et al.*, 1990; Herman *et al.*, 1992; Korecki *et al.*, 1997; Sadlin *et al.*, 1991; Tegze and Faigel, 1991, 1996). Theoretical and experimental set-ups have become possible not only for the holographic reconstruction of large macroscopic objects in the optical domain (Collier *et al.*, 1971; Gabor, 1948, 1949, 1951; Kogelnik, 1969) but also for the microscopic resolution and imaging of minute objects such as molecules and atoms. What makes this imaging possible is the advancement of the early holography (Collier *et al.*, 1971; Gabor, 1948,1949,1951; Kogelnik, 1969) first to the X-ray (Tegze and Faigel, 1991,1996) and γ-ray (Korecki *et al.*, 1997) domains and then to the generation and application of holograms by using matter waves such as electron emission from atoms (Barton, 1988; Han *et al.*, 1991; Harp *et al.*, 1990; Herman *et al.*, 1992; Sadlin *et al.*, 1991; Soroko, 2000; Spence and Koch, 2001; Szöke, 1986).

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In this work we wish to further, theoretically, expand and enlarge the diversity of waves used to record and reconstruct an initial subject. We use in this context gravitational waves which are so unique and different compared to electromagnetic and matter waves. First their interaction with matter is so weak that no direct[2] experimental set-up, even the giant LIGO (Abbott *et al.*, 2004), VIRGO (Acernese *et al.*, 2002), GEO (Danzmann, 1995) and TAMA (Ando and TAMA collaboration, 2002) facilities have succeeded up to now to directly detect them (see, for example a null result report (Abbott *et al.*, 2004) of a mutual search of LIGO and GEO for GW from the pulsar J1939+2134). Second, these waves, which are "ripples of curvature" (Misner *et al.*, 1973), influence the space-time through which they proceed by further curving it so as to increase or decrease the interval between the geodesics travelled by test particles (Misner *et al.*, 1973). Thus, holography which is thought to result from the diffraction and changes of the form of the passing waves by solid spatial objects may be discussed also from the point of view as if these diffraction and form changes result from passing through a region of spacetime in which the curvature is stronger than other regions (see Fig. 2).

The former discussion leads to the realization that one may diffracts and changes the form of an EMW passing through some finite region of space-time by two equivalent ways; (1) by some solid object placed in this region and (2) by a strong curvature (stronger than its values in neighbouring regions) imprinted in this region by GW. Moreover, one may, theoretically, obtain very similar diffractions for these two cases if he can adjust the external form of the spatial object to be such that an EMW which encounters it will be changed in the same manner as if it have passed through the region of strong curvature. Note that the principle of finite region with stronger curvature than other regions stands at the basis of all the different efforts made for detecting GW from the early mechanical bars of Weber (Weber, 1960,1969,1970) to the later mentioned interferometric detectors (Misner *et al.*, 1973).

Thus, one may discuss holography, as done here, by considering regions of stronger curvature (compared to neighbouring regions) without having to place in these regions any solid spatial object. We note in this context that already in the early holography (Collier *et al.*, 1971) the presence of a realistic solid objects were thought in certain cases to be unnecessary for recording real holograms such as, for example, in the computer-generated holograms (Brown and Lohmann, 1969; Collier *et al.*, 1971; Lesem *et al.*, 1969).

We must note that we neither try here to find the most general and complete theory of possible GW holography nor to discuss the fundamental aspects of the gravitational field as, for example, done in the works of

[2] Gravitaional waves were proved to exist by Taylor and Hulse (which receive the Nobel price in 1993 for this discovery) through indirect astronomical observations using radio telescope that measure the spiraling rate of two neighbouring neutron stars.

Finkelstein *et al* (see, for example (Finkelstein and Gibbs, 1993) and http://www.physics.gatech.edu/qr/papers.htm). We use the remarked property emphasized in (Misner *et al*., 1973) that the GW, under certain conditions, is indistinguishable from the EMW to also discuss, at least theoretically, a possible GW holography. For this it is sufficient to discuss plane GW's in the simplified transverse-traceless (TT) gauge (Misner *et al*., 1973) where these waves are purely spatial (Misner *et al*., 1973).

Thus, following the conventional holography (Collier *et al*., 1971), which necessitates a second reference wave which do not touch the solid object, we assume here another GW, denoted R, which do not pass through the region passed by the former GW (called S for subject) and serves as a reference to S. The two waves S and R are supposed to meet and interfere in a second space-time region which serves as a hologram just as the subject and reference waves in optical holography meet and interfere in the hologram.

Also, as in optical hologram (Collier *et al*., 1971) one may assume that the gravitational hologram is formed by the exposure (interference (Jenkins and White, 1976) of S and R) and the duration of it. But in contrast to the holograms recorded by EMW which are solid spatial objects made by altering, through exposure, the transmission or absorption properties of the recorded materials (Collier *et al*., 1971) (and include the plane and volume photosensitive and photographic emulsion holograms (Collier *et al*., 1971)) here the hologram can not be a similar solid object which records the interference of S and R. This is because, as mentioned, the effect of any GW is to increase space-time curvature (Misner *et al*., 1973) and this is, naturally, imprinted and recorded in the space-time itself (and not in any solid 3-dimensional object in it) so that a wave (EMW) passing through this region is difracted. Thus, the related hologram is, actually, a finite space-time region which records the interference between the GW's S and R so that if, as in the usual holography, the reference wave R is later sent again through this region, as illuminator, one reconstructs the space-time changes made by the subject wave S in its original region. We, theoretically, show that this is, actually, the case.

We note in this context that, in contrast to optical holography which records and reconstructs a 3-dimensional solid object where the temporal evolution is generally everaged and neglected (Collier *et al*., 1971) the case for the later microscopic holography is different. This is so, especially, for matter waves such as electrons which must be discussed in quantum terms (Ayman *et al*., 2001) for which time evolution is very important (Schiff, 1968). It has been shown, for example, in (Soroko, 2000) that the discussion of holograms made by matter waves has effects similar to those resulting from volume holograms (Collier *et al*., 1971; Kogelnik, 1969) except for replacing the spatial third dimension with the time variable. This emphasis of the time evolution is, especially, valid for the holography discussed here where, as described, the passing GW acts directly on the space-time medium itself and not on any spatial object in it. We, therefore,

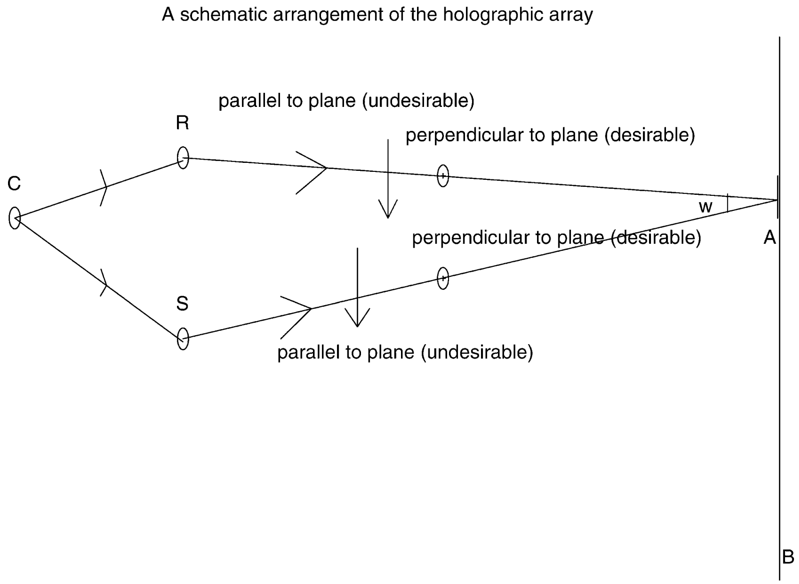

Fig. 1. The subject and reference waves are shown as rays originating at their common source at C from there they advance first to their respective points S and R and then to the small area A. It is also shown, for each wave, the two perpendicular components which are parallel and perpendicular to the plane of the figure and which denote the directions of polarization. The undesirable and desirable components of these polarizations (see text) are also shown.

emphasize these temporal changes and assume that the spatial components of the finite space-time region in which the waves S and R meet and interfere is very small (the small thin area A in Fig. 1).

In Section II we use the linearized weak field theory and introduce the relevant subject and reference GW together with their appropriate polarization components. In Section III we calculate the relevant intensities and the exposure. In Section IV we represent the hologram transmittance over the small area A and calculate the required reconstructed wave which will be found to be proportional to the original subject wave S. We conclude and summarize the obtained results in a Concluding Remarks Section. Also, since GW are, as mentioned, indistinguishable, under certain conditions, from EMW we use some known optical coherence expressions (Born and Wolf, 1964) which are introduced in a separate Appendix.

2. THE SUBJECT AND REFERENCE GW AND THEIR POLARIZATIONS

Figure 1 shows a schematic representation of the arrangement used in this discussion. In this set-up array we assume an initial GW which have been detected at the point C maybe by one or some collaboration of the mentioned interferometric detectors. This wave may be assumed to be divided, through a future technology, into two components which propagate to the two different regions denoted in Fig. 1 as S and R. From these two regions the relevant GW's, denoted also as S and R, propagate to the small region A where they meet and interfere. As mentioned, we use the linearized weak field approximation (Bergmann, 1976; Misner *et al.*, 1973) of general relativity which although refers to the surrounding space-time as if it were flat (as in special relativity) it, nevertheless, discuss experiments and their evolutions in a curved space-time formalism. In this theory the metric tensor components are given by (Misner *et al.*, 1973)

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O([h_{\mu\nu}]^2), \tag{1}$$

where $\eta_{\mu\nu}$ is the Lorentz metric of special relativity (Bergmann, 1976; Misner *et al.*, 1973) and $h_{\mu\nu}$ is a small perturbation. This $h_{\mu\nu}$ is identified with GW (Misner *et al.*, 1973; Thorne, 1980) which is itself a propagating perturbation of space-time (Misner *et al.*, 1973; Thorne, 1980). As known (Misner *et al.*, 1973), one of the simplest gauges to which one may subject the tensor $h_{\mu\nu}$ is the transverse-traceless gauge (TT) in which $h_{\mu\nu}$ has the smallest number of components (Misner *et al.*, 1973; Thorne, 1980). This is because in this gauge (Misner *et al.*, 1973; Thorne, 1980) $h_{\mu\nu}$ is purely spatial so $h_{0\mu} = 0$ and it is also transverse to the direction of its propagation so $h_{ij,j} = 0$. Its tracelessness introduces the additional condition of $h_{jj} = 0$. Thus, the gravitational wave is traditionally signified (Misner *et al.*, 1973) as $h^{TT}_{\mu\nu}$ which is the tensor $h_{\mu\nu}$ in the TT gauge. We take into account that $h^{TT}_{\mu\nu}$ is, as mentioned, purely spatial so we follow the traditional holographic notation and denote the relevant subject and reference waves by $S(x, y, z, t)$ and $R(x, y, z, t)$ respectively.

We assume that $S(x, y, z, t)$ and $R(x, y, z, t)$ are plane waves propagating along the respective vectors $\mathbf{n}_s$ and $\mathbf{n}_r$ and denote the two orthogonal directions which are perpendicular to $\mathbf{n}_s$ by $\mathbf{e}_{s_1}$ and $\mathbf{e}_{s_2}$ and those perpendicular to $\mathbf{n}_r$ by $\mathbf{e}_{r_1}$ and $\mathbf{e}_{r_2}$. Thus, following the notation in (Misner *et al.*, 1973) (where the discussion there refers to propagation along the z axis (see Chapter 35 there)) we denote the two unit linear polarization tensors of $S(x, y, z, t)$ as $\mathbf{e}_{+_s}$, $\mathbf{e}_{\times_s}$ and those of $R(x, y, z, t)$ as $\mathbf{e}_{+_r}$, $\mathbf{e}_{\times_r}$ and write

$$\begin{aligned} \mathbf{e}_{+_s} &= \mathbf{e}_{s_1} \otimes \mathbf{e}_{s_1} - \mathbf{e}_{s_2} \otimes \mathbf{e}_{s_2}, \quad \mathbf{e}_{x_s} = \mathbf{e}_{s_1} \otimes \mathbf{e}_{s_2} + \mathbf{e}_{s_2} \otimes \mathbf{e}_{s_1} \\ \mathbf{e}_{+_r} &= \mathbf{e}_{r_1} \otimes \mathbf{e}_{r_1} - \mathbf{e}_{r_2} \otimes \mathbf{e}_{r_2}, \quad \mathbf{e}_{x_r} = \mathbf{e}_{r_1} \otimes \mathbf{e}_{r_2} + \mathbf{e}_{r_2} \otimes \mathbf{e}_{r_1} \end{aligned} \tag{2}$$

where $\otimes$ is the tensor product. Note that each GW have, like its EMW analogue, two polarizations. Thus, if, for example, the propagating GW advances vertically through an interferometric detector then one polarization, actually, describes the known tidal forces (Misner *et al.*, 1973) which oscillate along the directions (Thorne, 1980) of east-west and north-south. The other polarization describes those tidal forces which oscillate along the directions (Thorne, 1980) of northeast-southwest and northwest-southeast. In the following we assume the subject and reference waves to be polychromatic so their sources emit light at several frequencies. We denote by $\mathbf{r}$ the position vector of a point in space and signify the respective direction cosines of $\mathbf{n}_s$, $\mathbf{n}_r$ by $\cos(\alpha_s)$, $\cos(\beta_s)$, $\cos(\gamma_s)$ and $\cos(\alpha_r)$, $\cos(\beta_r)$, $\cos(\gamma_r)$. Thus, taking into account that $k = \frac{2\pi}{\lambda}$ and defining the spatial frequencies $\xi_s = \frac{\cos(\alpha_s)}{\lambda}$, $\eta_s = \frac{\cos(\beta_s)}{\lambda}$, $\zeta_s = \frac{\cos(\gamma_s)}{\lambda}$, $\xi_r = \frac{\cos(\alpha_r)}{\lambda}$, $\eta_r = \frac{\cos(\beta_r)}{\lambda}$, $\zeta_r = \frac{\cos(\gamma_r)}{\lambda}$ one may write, for example, the subject and reference GW as

$$\begin{aligned} S(x, y, z, t) &= \Re\big[(A_{+_s}\mathbf{e}_{+_s} + A_{\times_s}\mathbf{e}_{\times_s})e^{ik\mathbf{r}\cdot\mathbf{n}_s}(c_0 e^{i2\pi f t} \\ &\quad + c_1 e^{i2\pi(f+\epsilon_1)t} + c_2 e^{i2\pi(f+\epsilon_2)t} + \cdots)\big] \\ &= \Re\Big[(A_{+_s}\mathbf{e}_{+_s} + A_{\times_s}\mathbf{e}_{\times_s})\exp[ik(x\cos(\alpha_s) \\ &\quad + y\cos(\beta_s) + z\cos(\gamma_s)]e^{i2\pi f t}\sum_i c_i e^{i2\pi\epsilon_i t}\Big] \\ &= \Re[(A_{+_s}\mathbf{e}_{+_s} + A_{\times_s}\mathbf{e}_{\times_s})\cdot \\ &\quad \times \exp[i2\pi(\xi_s x + \eta_s y + \zeta_s z)]e^{i2\pi f t}\cdot\mathbf{g}(t)] \end{aligned} \tag{3}$$

$$\begin{aligned} R(x, y, z, (t+\tau)) &= \Re\big[(A_{+_r}\mathbf{e}_{+_r} + A_{\times_r}\mathbf{e}_{\times_r})e^{ik\mathbf{r}\cdot\mathbf{n}_r}\big(c_0 e^{i2\pi f(t+\tau)} \\ &\quad + c_1 e^{i2\pi(f+\epsilon_1)(t+\tau)} + c_2 e^{i2\pi(f+\epsilon_2)(t+\tau)} + \cdots\big)\big] \\ &= \Re\Big[(A_{+_r}\mathbf{e}_{+_r} + A_{\times_r}\mathbf{e}_{\times_r})\exp[ik(x\cos(\alpha_r) + y\cos(\beta_r) \\ &\quad + z\cos(\gamma_r)]\cdot e^{i2\pi f(t+\tau)}\sum_i c_i e^{i2\pi\epsilon_i(t+\tau)}\Big] \\ &= \Re\big[(A_{+_r}\mathbf{e}_{+_r} + A_{\times_r}\mathbf{e}_{\times_r})\cdot\exp[i2\pi(\xi_r x \\ &\quad + \eta_r y + \zeta_r z)]\cdot e^{i2\pi f(t+\tau)}\cdot\mathbf{g}(t+\tau)\big] \end{aligned} \tag{4}$$

where $\Re$ denotes the real part of the following complex expression. The amplitudes A_{+_s}, $A_{\times_s}$ and A_{+_r}, $A_{\times_r}$ refer respectively to the modes of polarizations $\mathbf{e}_{+_s}$, $\mathbf{e}_{\times_s}$ and e_{+_r}, $e_{\times_r}$. In the following, for ease of notation, we denote $s_0 = A_{+_s}\mathbf{e}_{+_s} + A_{\times_s}\mathbf{e}_{\times_s}$ and $r_0 = A_{+_r}\mathbf{e}_{+_r} + A_{\times_r}\mathbf{e}_{\times_s}$. The parameter τ in Equation (4) is defined by $c\tau$, where c is the velocity of the GW which is equal to the velocity of light, so that $c\tau$ is the path difference between the paths of S and R as they propagate from their places at S and R (see Fig. 1) to the small area A. At the last results of Eqs. (3)–(4) we have used the definitions $\mathbf{g}(t) = \sum_i c_i e^{i2\pi\epsilon_i t}$ and $\mathbf{g}(t+\tau) = \sum_i c_i e^{i2\pi\epsilon_i(t+\tau)}$ where we assume that since S and R have common source (represented by the point C in Fig. 1) the quantities ϵ_i and the coefficients c_i are the same in $\mathbf{g}(t)$ and $\mathbf{g}(t+\tau)$.

The subject wave S from Equation (3) may be decomposed into a component, denoted $\mathbf{S}_=$, which is polarized parallel to the polarization direction of the reference wave R and another components, denoted S_+, which is perpendicular to this direction. Thus, denoting the angle between the polarization directions of S and R from Eqs. (3)–(4) by W (which is the same as that between the propagating rays S and R (see Fig. 1 and the text after Eqs. (5)–(6)) one may write $S_=$ and S_+ as

$$S_= = \Re[s_0 \cdot \exp[i(2\pi\xi_s x + 2\pi\eta_s y + 2\pi\zeta_s z)]e^{i2\pi ft} \cdot \mathbf{g}(t) \cdot \cos(W)] \tag{5}$$

$$S_+ = \Re[s_0 \cdot \exp[i(2\pi\xi_s x + 2\pi\eta_s y + 2\pi\zeta_s z)]e^{i2\pi ft} \cdot \mathbf{g}(t) \cdot \sin(W)] \tag{6}$$

In Figure 1 we show not only the propagating GW's of S and R but also the two components, for each GW, which are parallel and perpendicular to the plane of the figure. These components serve as polarization vectors. Note, however, that the angle between the polarization components of S and R which are parallel to the plane of Fig. 1 is W (which equals the angle between the propagating S and R (see Fig. 1)) whereas the angle between the polarization components perpendicular to this plane is zero. Thus, refering to the later components one may realize from Equations (5)–(6) that the component S_+ is zero whereas $S_=$ is maximum. In other words, the desirable components of polarization are those perpendicular to Fig. 1 as written explicitly in this figure. Note that this criterion applies also for optical holography (Collier *et al.*, 1971). We continue to use in the text the angle W since we are, especially, interested in the intensities of the GW and for this, as realized from the following section, one may obtain the same result regardless if he uses the perpendicular or the parallel components of polarization.

The effect of the polarizing tensors of either the gravitational plane wave S or R upon the space-time medium is best understood from Fig. 2 which, actually, shows the left half of Figure 35.2 in (Misner *et al.*, 1973). This figure shows how a closed circular (elliptic) array of test particles are changed, due to the resulting increased curvature, to elliptic (circular) array. These changes, as seen from the figure, are periodic and their exact form depend upon the values of the phase (shown in degrees at the right hand side the figure) and upon the unit linear polarization tensor.

3. THE INTENSITIES AND EXPOSURE OF THE GW

The separate intensities of the waves $R(x, y, (t+\tau))$, $S_=$ and S_+, denoted $I_R(t+\tau)$, $I_{S_=}(t)$ and $I_{S_+}(t)$, at the small area A are found from Eqs. (4), (5)–(6) as follows

$$I_R(t+\tau) = \langle R(x, y, (t+\tau))R^*(x, y, (t+\tau))\rangle = r_0^2\langle \mathbf{g}(t+\tau)\mathbf{g}^*(t+\tau)\rangle \tag{7}$$

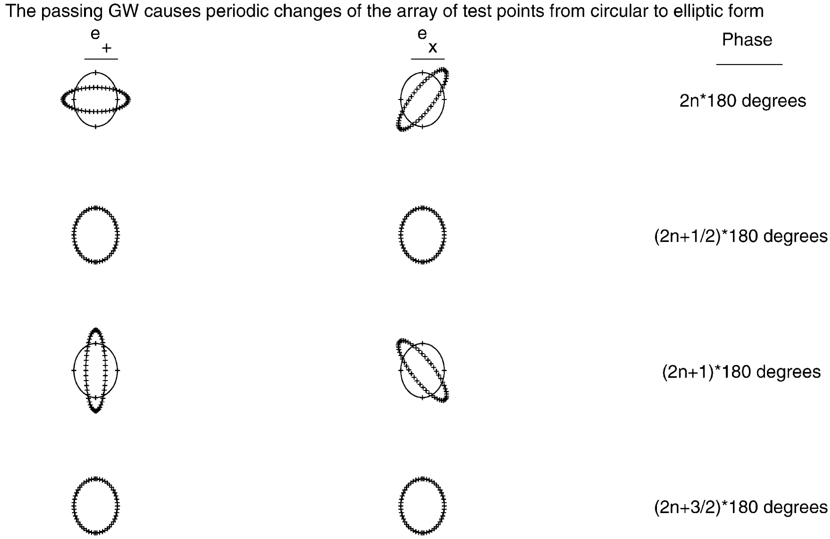

Fig. 2. The figure shows the effect of a GW passing through a region in which some test particles are shown arrayed in a closed form. The presence of the GW causes the space-time in this region to be more curved than usually when it is absent and this in turn changes the closed form of the array of test particles from a circular (elliptic) to an elliptic (circular) form. This behaviour, for the gravitational plane wave discussed here, is repeated in a periodic fashion and depends upon the values assumed by the phase as shown at the right (in degrees) and upon the corresponding nature of the polarization tensor e_x or e_+.

$$I_{S_=}(t) = \langle S_=(t)S_=^*(t)\rangle = s_0^2\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle \cos^2(W) \tag{8}$$

$$I_{S_+}(t) = \langle S_+(t)S_=^*(t)\rangle = s_0^2\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle \sin^2(W) \tag{9}$$

The overal intensity at the area A of the waves $R(x, y, (t+\tau))$, $S_=$, and S_+ is the sum of the separate intensities from Eqs. (7)–(9) plus the interference formed by these waves. But we must note that no interference is formed from S_+ and the polarization vector of $R(x, y, (t+\tau))$ because, as mentioned, they are perpendicular to each other. Thus, for interference we should take only the interaction of $S_=$ and the polarization direction of $R(x, y, (t+\tau))$ which are parallel to each other. In other words, the required total intensity at the small area A is

$$\begin{aligned} I_{\text{total}} &= I_R(t+\tau) + I_{S_=}(t) + I_{S_+}(t) + 2\Re[\langle R(x, y, (t+\tau))S_=^*\rangle] \\ &= r_0^2\langle \mathbf{g}(t+\tau)\mathbf{g}^*(t+\tau)\rangle + s_0^2\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle \cos^2(W) + s_0^2\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle \\ &\quad \times \sin^2(W) + 2\Re[r_0 s_0 \cos(W)\cdot \exp[i2\pi((\xi_r-\xi_s)x \\ &\quad + (\eta_r-\eta_s)y + (\zeta_r-\zeta_s)z)]\cdot e^{i2\pi f\tau}\langle \mathbf{g}(t+\tau)\mathbf{g}^*(t)\rangle] \end{aligned} \tag{10}$$

From Eq. ($A7$) in the Appendix one may realize (Collier *et al.*, 1971) that since $|\hat{\mu}_T(\tau)| = |\mu_T(\tau)|$ where $\hat{\mu}_T(\tau) = \mu_T(\tau)e^{-i2\pi f\tau} = \frac{\langle \mathbf{g}(t+\tau)\mathbf{g}^*(t)\rangle}{\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle}$ it is possible to write $\hat{\mu}_T(\tau) = |\mu_T(\tau)| \cdot e^{i\zeta(\tau)}$ where $e^{i\zeta(\tau)}$ is a phase factor. Thus, using the last equation one may write the total intensity from Eq. (10) as (Collier *et al.*, 1971)

$$\begin{aligned} I_{\text{total}} &= r_0^2\langle \mathbf{g}(t+\tau)\mathbf{g}^*(t+\tau)\rangle + s_0^2\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle + 2\Re[r_0 s_0 \cos(W) \\ &\quad \times \exp[i2\pi((\xi_r - \xi_s)x + (\eta_r - \eta_s)y + (\zeta_r - \zeta_s)z)]e^{i2\pi f\tau}|\mu_T(\tau)| \cdot e^{i\zeta(\tau)} \\ &\quad \times \langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle] = r_0^2\langle \mathbf{g}(t+\tau)\mathbf{g}^*(t+\tau)\rangle + s_0^2\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle \\ &\quad + 2r_0 s_0 \cos(W) \cdot |\mu_T(\tau)| \cos(\beta(x,y,z,\tau))\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle, \end{aligned} \tag{11}$$

where $\beta(x,y,z,t) = 2\pi[(\xi_r - \xi_s)x + (\eta_r - \eta_s)y + (\zeta_r - \zeta_s)z + f\tau] + \zeta(\tau)$. The intensity from Eq. (11) is recorded on the hologram through exposure E which is assumed to be, in analogy with optical holograms, proportional (Collier *et al.*, 1971) to the product of the intensity I_{total} and the exposure time τ_e. That is, denoting the proportionality constant by C one may write, using Eq. (11), the exposure as

$$\begin{aligned} E(x,y,z,t) = C\tau_e I_{\text{total}} &= C\tau_e[s_0^2\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle + r_0^2\langle \mathbf{g}(t+\tau)\mathbf{g}^*(t+\tau)\rangle \\ &\quad + 2r_0 s_0 \cos(W)|\mu_T(\tau)|\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle \cos(\beta(x,y,z,\tau))] \\ &= E_0 + E_1(x,y,z,t), \end{aligned} \tag{12}$$

where

$$\begin{aligned} E_0 &= C\tau_e\big(s_0^2\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle + r_0^2\langle \mathbf{g}(t+\tau)\mathbf{g}^*(t+\tau)\rangle\big) \\ E_1(x,y,z,t) &= 2C\tau_e s_0 r_0 \cos(W)|\mu_T(\tau)|\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle \cos(\beta(x,y,z,\tau)) \end{aligned}$$

4. THE HOLOGRAM TRANSMITTANCE AND THE RECONSTRUCTED GW

Referring to the former equations one may realize that if the expression $\frac{r_0^2}{s_0^2}$ satisfies $\frac{r_0^2}{s_0^2} > 1$ over the small area A then one also have $\frac{r_0^2}{s_0^2} > \frac{r_0}{s_0}$ and consequently $E_0 > E_1$ over this area A of the hologram. In such case, analogously to optical holography (Collier *et al.*, 1971), one may write the hologram transmittance over the area A as a Taylor series (Collier *et al.*, 1971)

$$\mathbf{t}_E = \mathbf{t}_E(E_0) + E_1\frac{d\mathbf{t}_E}{dE}\bigg|_{E_0} + \frac{1}{2}E_1^2\frac{d^2\mathbf{t}_E}{dE^2}\bigg|_{E_0} + \cdots \tag{13}$$

In optical holography this representation of the transmittance is general and includes the possibility of either amplitude or phase modulation by the hologram (Collier *et al.*, 1971). Now, (1): we assume that all the coefficients of second and higher order terms in Eq. (13) $\frac{d^2\mathbf{t}_E}{dE^2}|_{E_0}$, $\frac{d^3\mathbf{t}_E}{dE^3}|_{E_0}$, ... are negligible and (2): that the factor $\cos(\beta(x, y, z, \tau))$ from Eq. (12) is written as a sum of exponentials from which the term $\frac{1}{2}e^{-i\beta(x,y,z,\tau)}$ is chosen (as done in optical holography (Collier *et al.*, 1971)) where β is given by the inline equation after Eq. (11). Thus, for reconstructing the subject wave $S(x, y, z, t)$ from Eq. (3) one illuminates the hologram $\mathbf{t}_E$ with the reference wave $R(x, y, z, (t+\tau))$ from Equation (4) so that the GW obtained from the exposure E_1 (the constant E_0 has no role in this reconstruction) at the small area A is

$$\begin{aligned} W_r(x, y, z, t) &= R(x, y, z, (t+\tau))\mathbf{t}_E = r_0 \cdot \exp[i(2\pi(\xi_r x + 2\pi\eta_r y \\ &\quad + 2\pi\zeta_r z))] \cdot e^{i2\pi f(t+\tau)}\mathbf{g}(t+\tau)E_1\frac{d\mathbf{t}_E}{dE}|_{E_0} \\ &= C\tau_e r_0^2 s_0 \cos(W)|\mu_T(\tau)|\langle\mathbf{g}(t)\mathbf{g}^*(t)\rangle \cdot e^{-i(\beta(x,y,z,\tau))} \cdot \\ &\quad \times \exp[i(2\pi(\xi_r x + 2\pi\eta_r y + 2\pi\zeta_r z))] \cdot e^{i2\pi f(t+\tau)}\mathbf{g}(t+\tau)\frac{d\mathbf{t}_E}{dE}|_{E_0} \\ &= C\tau_e r_0^2 s_0 \cos(W)|\mu_T(\tau)|\langle\mathbf{g}(t)\mathbf{g}^*(t)\rangle \\ &\quad \times \exp[i2\pi(\xi_s x + \eta_s y + \zeta_s z)]e^{-i\zeta(\tau)} \cdot e^{i2\pi ft}\mathbf{g}(t+\tau)\frac{d\mathbf{t}_E}{dE}|_{E_0} \end{aligned} \tag{14}$$

In order to continue in our analytical reconstruction of the subject wave $S(x, y, z, t)$ we first show that $\mathbf{g}(t+\tau)\cdot\langle\mathbf{g}(t)\mathbf{g}^*(t)\rangle = \mathbf{g}(t)\cdot\langle\mathbf{g}(t+\tau)\mathbf{g}^*(t)\rangle$. This is done by taking into account that $\mathbf{g}(t) = \sum_i c_i e^{i2\pi\epsilon_i t}$, $\quad \mathbf{g}(t+\tau) = \sum_i c_i e^{i2\pi\epsilon_i(t+\tau)}$ (see the discussion after Equation. (4)) and $\langle\mathbf{g}(t)\mathbf{g}^*(t)\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\mathbf{g}(t)\mathbf{g}^*(t)dt$, $\langle\mathbf{g}(t+\tau)\mathbf{g}^*(t)\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\mathbf{g}(t+\tau)\mathbf{g}^*(t)dt$ (see Eq. ($A5$) in the Appendix). Thus, integrating the elementary exponentials and taking the corresponding limits $T\to\infty$ one obtains the following results

$$\langle\mathbf{g}(t)\mathbf{g}^*(t)\rangle = \sum_i |c_i|^2, \quad \langle\mathbf{g}(t+\tau)\mathbf{g}^*(t)\rangle = \sum_i |c_i|^2 e^{i2\pi\epsilon_i\tau} \tag{15}$$

From the last equations, the definitions of $\mathbf{g}(t)$, $\mathbf{g}(t+\tau)$ and the discussion after Eq. (4) about ϵ_i and c_i which are the same in $\mathbf{g}(t)$ and $\mathbf{g}(t+\tau)$ one may realize that

$$\begin{aligned} \mathbf{g}(t+\tau)\cdot\langle\mathbf{g}(t)\mathbf{g}^*(t)\rangle &= \sum_i |c_i|^2 e^{i2\pi\epsilon_i(t+\tau)} \cdot \sum_i |c_i|^2 \\ &= \sum_i |c_i|^2 e^{i2\pi\epsilon_i t} \cdot \sum_i e^{i2\pi\epsilon_i\tau}|c_i|^2 \\ &= \mathbf{g}(t)\cdot\langle\mathbf{g}(t+\tau)\mathbf{g}^*(t)\rangle, \end{aligned} \tag{16}$$

which is what we set to prove. Using the last equation and Eq. $(A7)$ in the Appendix (from which we see, as mentioned after Eq. (10), that the equality $|\hat{\mu}_T(\tau)| = |\mu_T(\tau)|$ leads to $\hat{\mu}_T(\tau) = |\mu_T(\tau)| \cdot e^{i\zeta(\tau)}$) one may write the reconstructed wave from Equation (14) as

$$
\begin{aligned}
W_r(x, y, z, t) &= C\tau_e r_0^2 s_0 \cos(W)|\mu_T(\tau)|\langle \mathbf{g}(t+\tau)\mathbf{g}^*(t)\rangle \exp[i2\pi(\xi_s x \\
&\quad + \eta_s y + \zeta_s z)] \cdot e^{i2\pi ft} e^{-i\zeta(\tau)} \cdot \mathbf{g}(t)\frac{d\mathbf{t}_E}{dE}|_{E_0} \\
&= C\tau_e r_0^2 s_0 \cos(W)|\mu_T(\tau)|^2 \cdot \langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle e^{i2\pi ft} \\
&\quad \times \exp[i2\pi(\xi_s x + \eta_s y + \zeta_s z)] \cdot \mathbf{g}(t)\frac{d\mathbf{t}_E}{dE}|_{E_0}
\end{aligned} \tag{17}
$$

Now, taking into account our neglection (see the discussion after Eq. (13)) of the coefficients of the higher order terms in Eq. (13) $\frac{d^2\mathbf{t}_E}{dE^2}|_{E_0}, \frac{d^3\mathbf{t}_E}{dE^3}|_{E_0}, \ldots$ one may realize that if $\frac{d^2\mathbf{t}_E}{dE^2}|_{E_0} = 0$ then $\frac{d\mathbf{t}_E}{dE}|_{E_0} = constant$. Thus, using the last result, the definition of s_0 as given after Eq. (4), and the first of Eq. (15) one may write the reconstructed wave from Eq. (17) as

$$
\begin{aligned}
W_r(x, y, z, t) &= constant \cdot s_0 \cdot \mathbf{g}(t)e^{i2\pi ft} \exp[i2\pi(\xi_s x + \eta_s y + \zeta_s z)] \\
&= constant \cdot (A_{+_s}\mathbf{e}_{+_s} + A_{\times_s}\mathbf{e}_{\times_s})\mathbf{g}(t)e^{i2\pi ft} \\
&\quad \times \exp[i2\pi(\xi_s x + \eta_s y + \zeta_s z)] \\
&= constant \cdot S(x, y, z, t),
\end{aligned} \tag{18}
$$

where $S(x, y, z, t)$ is the subject wave given by Eq. (3). Thus, as in optical holography, we see that the original subject wave has been reconstructed.

5. CONCLUDING REMARKS

We have discussed gravitational wave holography in which all the involved waves; subject, reference and illumnator are gravitational waves. The nature of these waves, compared to their electromagetic analogues, causes the resulting holography to be somewhat unique. First, the interaction of these waves with matter is so weak that no experimental set-up have, up to now, succeeded to directly[3] detect them. Second, these waves act upon the space-time structure itself by increasing its curvature so that any wave (for example, EMW) which passes in this region undergoes similar changes as those occuring when encountering a corresponding solid spatial object. That is, the same diffraction and form changes in some finite region may result from either a strong space-time curvature in it

[3] Gravitaional waves were proved to exist by Taylor and Hulse (which receive the Nobel price in 1993 for this discovery) through indirect astronomical observations using radio telescope that measure the spiraling rate of two neighbouring neutron stars.

compared to other neighbouring regions or from a corresponding suitably designed solid object. Note that this is reminiscent of the famous Einstein "elevator" (Bergmann, 1976) in which a man closed inside this accelerating cabin can not be sure if this accelaration is due to a gravitational force all around or maybe he is in a region absent of any gravitation and that other force pulls the cabin with the known attraction of gravity.

The strong curvature imprinted by the GW upon the space-time medium remains in this medium (Misner *et al.*, 1973) even after the wave have completely passed away (Misner *et al.*, 1973) especially if this GW is strong enough or if it has passed this region a large number of times (Misner *et al.*, 1973).

In our discussion we have used the known methods of optical holograph (Coyllier *et al.*, 1971; Gabor, 1948,1949,1951) and, especially, the realization (Misner *et al.*, 1973) that, under certain conditions such as very small wavelength, one can not, theoretically, diffentiate between GW and EMW. We have, thus, shown that passing a reference GW (not in the same region passed by the subject wave) and letting these two waves meet and interfere in some other space-time region (hologram) then if this reference wave is again passed, as the corresponding EMW illuminator, through this hologram the result will be a reconstruction of the subject wave.

Although this discussion is purely theoretical one may hope that a future advanced technology will be developed which will enable the next generation of scientists to use and manipulate GW the same way we are able now to use EMW.

APPENDIX

We use here the mentioned characteristic of the almost theoretical identity (valid under certain conditions such as very short wavelengths) between GW and EMW and assume, as we do in the main text, that we may use the known results (Born and Wolf, 1964) regarding the spatial and (or) temporal coherence between two waves. We, thus, introduce here some relevant expressions (Born and Wolf, 1964; Collier *et al.*, 1971) for the coherence between two complex electric waves $\mathbf{v}_1$ and $\mathbf{v}_2$ which advance from points P_1 and P_2 at some screen to the point Q at another. This is similar to the set-up in Fig. 1 in which the two GW's S and R propagate from the corresponding points S and R to the small area A. The time average of the interference term between $\mathbf{v}_1$ and $\mathbf{v}_2$ is written as (Born and Wolf, 1964)

$$\langle \mathbf{v}_1 \mathbf{v}^*{}_2 + \mathbf{v}_1^* \mathbf{v}_2 \rangle = 2\Re \langle \mathbf{v}_1 \mathbf{v}_2^* \rangle \tag{A1}$$

It has been shown in (Born and Wolf, 1964) that the time average from Equation ($A1$) can be expressed in terms of the complex degree of coherence $\Upsilon_{12}(\tau)$ which relates the correlation of $\mathbf{v}_1$ and $\mathbf{v}_2$ at P_1 and P_2 to the interference time average at Q. The parameter τ denotes the time difference in arrival from the points

P_1, P_2 to Q. Thus, denoting by $\mathbf{v}(t)_{P_1}$, $\mathbf{v}(t)_{P_2}$ the complex fields at P_1, P_2 and by $2\langle \mathbf{v}(t)_{P_1}\mathbf{v}^*(t)_{P_1}\rangle$, $2\langle \mathbf{v}(t)_{P_2}\mathbf{v}^*(t)_{P_2}\rangle$ the corresponding light intensities one may define the complex coherence $\Upsilon(\tau)$ (Born and Wolf, 1964) as

$$\begin{aligned}\Upsilon_{12}(\tau) &= \frac{\langle \mathbf{v}(t+\tau)_{P_1}\mathbf{v}^*(t)_{P_2}\rangle}{[\langle \mathbf{v}(t)_{P_1}\mathbf{v}^*(t)_{P_1}\rangle\langle \mathbf{v}(t)_{P_2}\mathbf{v}^*(t)_{P_2}\rangle]^{\frac{1}{2}}} \\ &= \frac{\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\mathbf{v}(t+\tau)_{P_1}\mathbf{v}^*(t)_{P_2}dt}{[(\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\mathbf{v}(t)_{P_1}\mathbf{v}^*(t)_{P_1}dt)(\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\mathbf{v}(t)_{P_2}\mathbf{v}^*(t)_{P_2}dt)]^{\frac{1}{2}}}\end{aligned} \tag{A2}$$

The two Equations $(A1)-(A2)$ were related in (Born and Wolf, 1964) through

$$2\Re[\langle \mathbf{v}_1\mathbf{v}_2^*\rangle] = 2(I_1I_2)^{\frac{1}{2}}\Re[\Upsilon_{12}(\tau)] = 2(I_1I_2)^{\frac{1}{2}}|\Upsilon_{12}(\tau)|\cos(\beta_{12}(\tau)), \tag{A3}$$

where I_1, I_2 are the intensities at Q from P_1, P_2 and $|\Upsilon_{12}(\tau)|$, $\beta_{12}(\tau)$ are the modulus and phase of $\Upsilon_{12}(\tau)$. Note that $\Upsilon_{12}(\tau)$ is a measure of both the temporal and spatial aspects of the coherence (Born and Wolf, 1964; Collier *et al.*, 1971). For $\tau = 0$ one may assume the points P_1, P_2 at screen A to be at the (x, y) plane where P_1 is located at the origin of this plane. Thus, denoting $\Upsilon_{12}(0) = \mu_s(x, y)$ one may write Equation $(A2)$ for this case as

$$\mu(x,y)_s = \frac{\int_{-\infty}^{\infty}\mathbf{v}(0,0,t)\mathbf{v}^*(x,y,t)dt}{\left[\int_{-\infty}^{\infty}\mathbf{v}(0,0,t)\mathbf{v}^*(0,0,t)dt\int_{-\infty}^{\infty}\mathbf{v}(x,y,t)\mathbf{v}^*(x,y,t)\,dt\right]^{\frac{1}{2}}} \tag{A4}$$

Note that now, in cotrast to $\Upsilon_{12}(\tau)$ which denotes both temporal and spatial coherence, $\mu(x, y)_s$ is the complex spatial coherence of the source in the (x, y) plane. Note also that if the waves $\mathbf{v}_{P_1}$, $\mathbf{v}_{P_2}$ propagate along the same direction from a point source then $\mathbf{v}(t)_{P_1} = \mathbf{v}(t)_{P_2}$ and $\mu(x, y)_s = 1$ (see Eq. $(A4)$). In such a case one may replace in Eq. $(A2)$ (Collier *et al.*, 1971) $\Upsilon_{12}(\tau)$ by $\mu_T(\tau)$, equate $\mathbf{v}(t)_{P_1} = \mathbf{v}(t)_{P_2} = \mathbf{v}(t)$, $\mathbf{v}(t+\tau)_{P_1} = \mathbf{v}(t+\tau)$ and write for $\mu_T(\tau)$ which is the complex temporal coherence

$$\mu_T(\tau) = \frac{\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\mathbf{v}(t+\tau)\mathbf{v}^*(t)\,dt}{\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\mathbf{v}(t)\mathbf{v}^*(t)\,dt} \tag{A5}$$

Substituting in the last equation $\mathbf{v}(t) = S(x, y, z, t)$, where $S(x, y, z, t)$ is given by Equation (3) one obtains

$$\mu_T(\tau) = e^{i2\pi f\tau}\cdot\frac{\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\mathbf{g}(t+\tau)\mathbf{g}^*(t)\,dt}{\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\mathbf{g}(t)\mathbf{g}^*(t)\,dt} = e^{i2\pi f\tau}\frac{\langle \mathbf{g}(t+\tau)\mathbf{g}^*(t)\rangle}{\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle} \tag{A6}$$

One may solve the last equation (Collier *et al.*, 1971) for $\frac{\langle \mathbf{g}(t+\tau)\mathbf{g}^*(t)\rangle}{\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle}$ and obtains

$$\frac{\langle \mathbf{g}(t+\tau)\mathbf{g}^*(t)\rangle}{\langle \mathbf{g}(t)\mathbf{g}^*(t)\rangle} = \mu_T(\tau)e^{-i2\pi f\tau} = \hat{\mu}_T(\tau) \tag{A7}$$

Equation $(A5)$ may be Fourier transformed into

$$\mu_T(\tau) = \frac{\int_{-\infty}^{\infty} \mathbf{V}(f)\mathbf{V}^*(f)\cdot e^{i2\pi f\tau}\, df}{\int_{-\infty}^{\infty} \mathbf{V}(f)\mathbf{V}^*(f)\, df}, \tag{A8}$$

where $\mathbf{V}(f)$ is the temporal Fourier transform of $\mathbf{v}(\tau)$ (Bracewell, 1965; Collier *et al.*, 1971).

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